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# Finite-size effects on the minimal conductivity in graphene with Rashba spin-orbit coupling

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Budapest, 16 October, 2015

# Motivation

Experiment

nature COMMUNICATIONS

ARTICLE

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Tunable Fermi level and hedgehog spin texture in gapped graphene

A. Varykhalov[1], J. Sánchez-Barriga[1], D. Marchenko[1,2], P. Hlawenka[1], P.S. Mandal[1] & O. Rader[1]

In graphene the
spin–orbit coupling is extremely weak
(intrinsic band splitting $\approx 10^{-5}\,\mathrm{eV}$)

Spin–orbit coupling can be enhanced by several orders of magnitude through contact of graphene to heavier elements

↓

Intercalation of Au atoms (monolayer) under graphene on Fe(110)

↓

Intercalation of Au atoms decouples the graphene from the Fe substrate

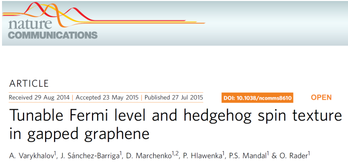
↓

Quasi-freestanding graphene

↓

Spin- and angle-resolved photoemission measurments

# Motivation: hedgehog spin texture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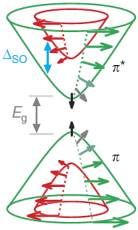
Tunable Fermi level and hedgehog spin texture in gapped graphene

A. Varykhalov[1], J. Sánchez-Barriga[1], D. Marchenko[1,2], P. Hlawenka[1], P.S. Mandal[1] & O. Rader[1]

Ingredients for novel spin texture:

- Giant Rashba spin–orbit (RSO) interaction
  (70 meV splitting away from the Dirac point)
- Breaking the six-fold graphene symmetry
  at the interface (graphene on Fe(110) substrate)

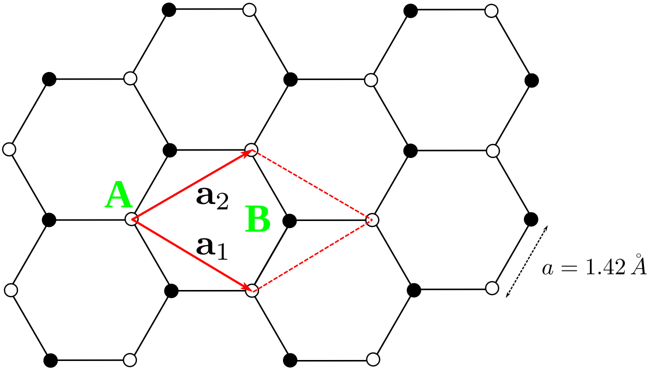
Formation of an out-of-plane
hedgehog-type
spin configuration




P. Rakyta, A. Kormányos & J. Cserti: *Effect of sublattice asymmetry and spin-orbit interaction on out-of-plane spin polarization of photoelectrons,* Phys. Rev. B **83**, 155439 (2011).

`Rakyta et al. predict the formation of a hedgehog-like spin texture at the gapped Dirac point.
This is exactly what is observed in our present experiment.'

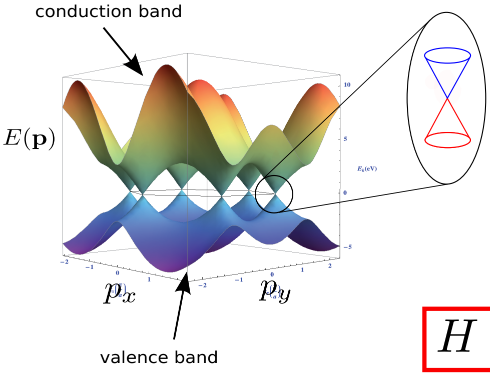
# Honeycomb lattice of carbon atoms in graphene

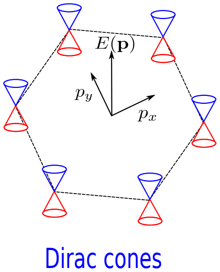



- single atomic layers of carbon atoms: graphene
- honey comb lattice
- distance between neighboring atoms: 1,42 Angström
- two sub-lattices ( atom **A** and **B**)

# Dispersion relation

D. P. DiVincenzo and E. J. Mele, Phys. Rev. B 29, 1685 (1984)




$$H = c\,\boldsymbol{\sigma}\boldsymbol{p}$$

$$c \approx c_0/300$$

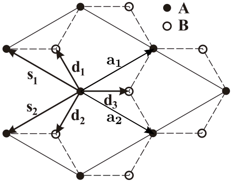
Linear dispersion around K.
Dirac cones. No gap at K.

Relativistic, zero mass, 2 dimensional electron

# Theoretical model for
# Rashba spin-orbit coupling in graphene
## Tight binding model

C.L. Kane, E.J. Mele, Phys. Rev. Lett. **95** (2005) 226801.
M. Zarea, N. Sandler, Phys. Rev. B **79** (2009) 165442.
M. Zarea, N. Sandler, New J. Phys. **11** (2009) 095014.
P. Rakyta, A. Kormányos, J. Cserti, Phys. Rev. B **82** (2010) 113405.

$$\begin{aligned} H_{TB} &= H_0 + H_R, \qquad \text{where} \\ H_0 &= -\gamma \sum_{\langle i,j\rangle,\sigma} \left( a^\dagger_{i\sigma} b_{j\sigma} + \text{h.c.} \right), \\ H_R &= i\,\lambda \sum_{\langle i,j\rangle,\mu,\nu} \left[ a^\dagger_{i\mu} \left( \mathbf{s}_{\mu\nu} \times \widehat{\mathbf{d}}_{\langle i,j\rangle} \right)_z b_{j\nu} - h.c. \right] \end{aligned}$$



# Theoretical model for Rashba spin-orbit coupling in graphene

## Continuum model

C.L. Kane, E.J. Mele, Phys. Rev. Lett. **95** (2005) 226801.
M. Zarea, N. Sandler, Phys. Rev. B **79** (2009) 165442.
M. Zarea, N. Sandler, New J. Phys. **11** (2009) 095014.
P. Rakyta, A. Kormányos, J. Cserti, Phys. Rev. B **82** (2010) 113405.

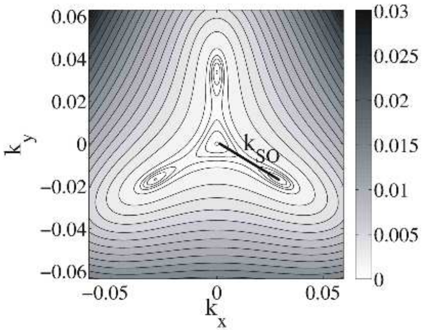
$$H_K = \begin{pmatrix} 0 & v_F p_- & 0 & v_\lambda p_+ \\ v_F p_+ & 0 & -3i\lambda & 0 \\ 0 & 3i\lambda & 0 & v_F p_- \\ v_\lambda p_- & 0 & v_F p_+ & 0 \end{pmatrix}$$

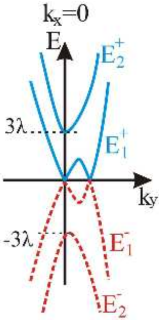
$$v_F = 3\gamma d/(2\hbar), v_\lambda = 3\lambda d/(2\hbar), p_\pm = p_x \pm i p_y$$

$p_x, p_y$ are momentum operators

# Energy bands near the K point

Trigonal warping:



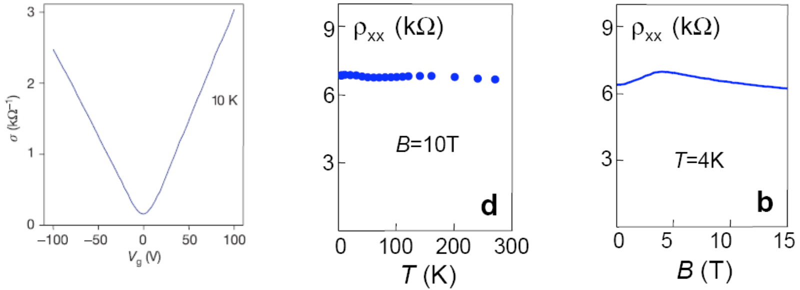




**Note**: monolayer graphene with RSO coupling is unitary equivalent to bilayer graphene without RSO interaction but including the trigonal warping effect due to interlayer hopping.

P. Rakyta, A. Kormányos, J. Cserti, Phys. Rev. B **82**, 113405 (2010)

# Minimal conductivity in single layer graphene

K. S. Novoselov, E. McCann, S. V. Morozov, V. I. Fal'ko, M. I. Katsnelson, U. Zeitler, D. Jiang, F. Schedin, A. K. Geim, Nature Physics **2**, 177 (2006)




Independent of temperature and magnetic field

$$\sigma^{\min} \approx 4\,\frac{e^2}{h}$$

# What states do contribute to the conduction?
# DOS

Dispersion relation: $$E_{\pm} = \pm \hbar c |\mathbf{k}|$$

DOS: $$\varrho(E) = \frac{2}{\pi^2} \frac{A_c}{\hbar^2 v^2} |E|$$

At the Dirac point (E = 0) the DOS is zero !!

# Evanescent modes

$E_\pm = \pm\hbar c|\mathbf{k}|$

At the Dirac point (E=0)

$$\Longrightarrow \quad 0 = |\mathbf{k}| = \sqrt{k_x^2 + k_y^2}$$

$\Downarrow$

if $k_y$ real

$\Downarrow$

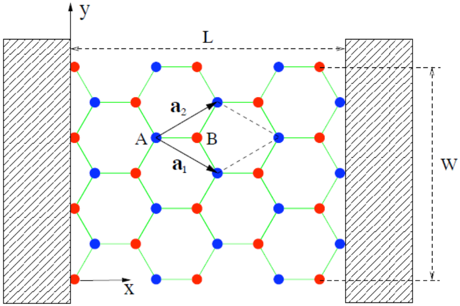
$k_x$ imaginary

$\Downarrow$

$e^{ik_x x}$ plane wave

$\Downarrow$

evanescent mode




The evanescent modes result in a finite conductivity!

for ballistic graphene
the minimal conductivity:

$$\sigma_{xx}^{\min} = \frac{4}{\pi}\frac{e^2}{h}$$

# Previous results for single layer graphene

**minimal conductivity:** $\sigma_{xx}^{\min} = (4/\pi)\, e^2/h$

- E. Fradkin, PRB **63**, 3263 (1986)
- A. W. W. Ludwig, M. P. A. Fisher, R. Shankar, and G. Grinstein, PRB **50**, 7526 (1994)
- P. A. Lee, PRL **71**, 1887 (1993)
- E. V. Gorbar, V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, PRB **66**, 045108 (2002)
- V. P. Gusynin and S. G. Sharapov, PRL **95**, 146801 (2005)
- N. M. R. Peres, F. Guinea, and A. H. Castro Neto, PRB **73**, 125411 (2006)
- M. I. Katsnelson, Eur. J. Phys B **51**, 157 (2006)
- J. Tworzydlo, B. Trauzettel, M. Titov, A. Rycerz, C.W.J. Beenakker, PRL **96**, 246802 (2006)

K. Ziegler, Phys. Rev. Lett. **97**, 266802 (2006) $\sigma_{xx}^{\min} = \pi\, e^2/h$

K. Nomura and A. H. MacDonald,
Phys. Rev. Lett. **98**, 076602 (2007)

$\sigma_{xx}^{\min} = (4/\pi)\, e^2/h$ ← Short range scattering

$\sigma_{xx}^{\min} = 4\, e^2/h$ ← Coulumb scattering

L. A. Falkovsky and A. A. Varlamov,
The European Physical Journal B **56**, 281 (2007) $\sigma_{xx}^{\min} = (\pi/2)\, e^2/h$

# Previous results for bilayer graphene

M. Koshino and T. Ando, Phys. Rev. B **73**, 245403 (2006)

$$\sigma_{xx}^{\min} = (8/\pi)\, e^2/h$$ ← strong-disorder regime

$$\sigma_{xx}^{\min} = (24/\pi)\, e^2/h$$ ← weak-disorder regime

M. I. Katsnelson, Eur. Phys. J. B **52**, 151-153 (2006)

$$\sigma_{xx}^{\min} = 2\, e^2/h$$

J. Cs., PRB **75**, 033405 (2007)
I. Snyman, C.W.J. Beenakker,
Phys.Rev.B **75**, 045322 (2007)

$$\sigma_{xx}^{\min} = (8/\pi)\, e^2/h$$

no trigonal warping

J. Cs., A. Csordás, and Gy. Dávid,
Phys. Rev. Lett. **99**, 066802 (2007)

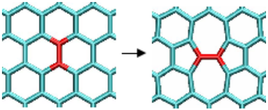
$$\sigma_{xx}^{\min} = (24/\pi)\, e^2/h$$

with trigonal warping

# Physical reasons for the minimal conductivity

**Disorder:**

- absorbed atoms, molecules (H, CH)
- vacancies
- topological defects, eg, Stone-Wales
- non perfect planes, ripples
- edge of the sample
- role of the SiO substrate

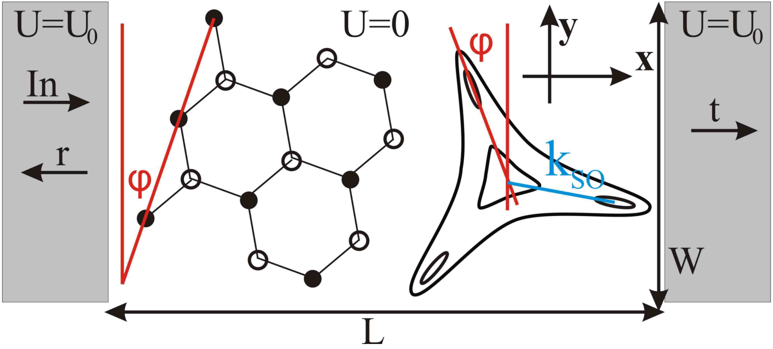


**Scattering processes:**

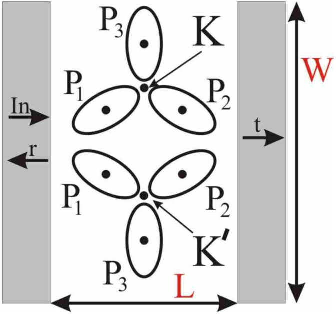
- short range scattering
- Coulumb scattering
- electron - phonon, electron-electron scattering

N. N. Peres: *Colloquium:* The transport properties of graphene: An introduction, Rev. Mod. Phys. **82**, 2673 (2010)

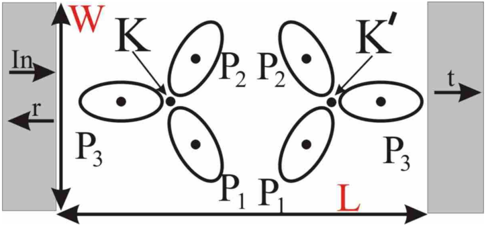
# Geometry for our calculations
# Finite size and edge effects



# Armchair and zig-zag edges




Armchair edge




Zig-zag edge

# Landauer-Büttiker formula

Tight binding & continuum model:

$$G = \frac{e^2}{h} \sum_{m,n} |t_{mn}|^2$$

Transmission amplitude

Conductivity in continuum model:

$$\sigma = 2\,\frac{L}{W}\,G = \frac{\sigma_0}{4}\,L \int_{-\infty}^{\infty} \mathrm{d}q \sum_{m,n} |t_{mn}(q)|^2$$

Integration over the transverse wave numbers

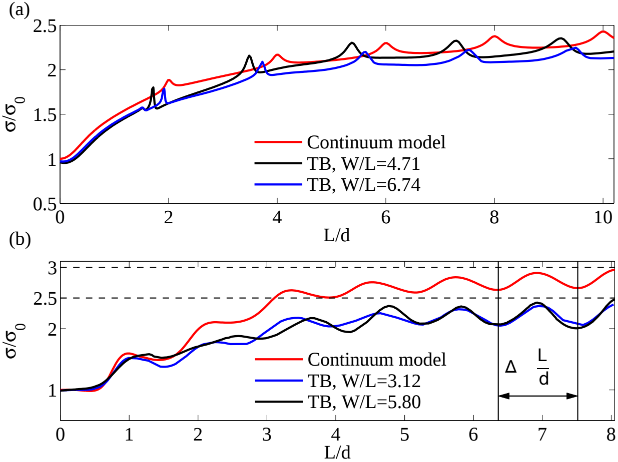
$$\sigma_0 = \frac{4}{\pi}\frac{e^2}{h}$$

# Results: conductivity for zig-zag and armchair edges

(TB and continuum model)

Conductivity as a function of the length of the sample:

units: $\sigma_0 = \frac{4}{\pi}\frac{e^2}{h}$




$\sigma = 7/3\sigma_0$

(TB & cont.)

Zig-zag

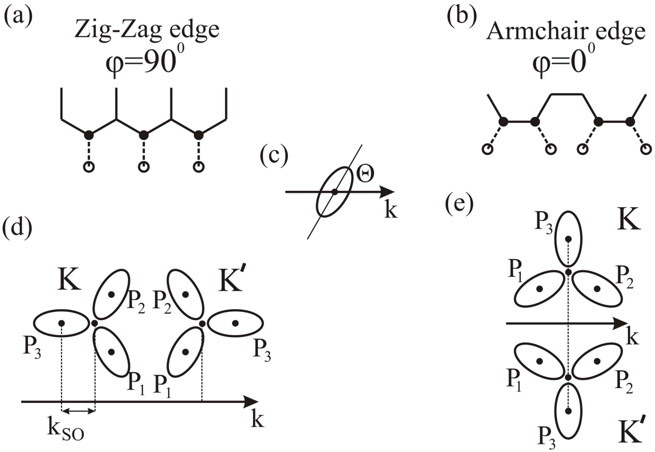
$\sigma = 3\sigma_0$ (cont.)

$\sigma = 5/2\sigma_0$ (TB)

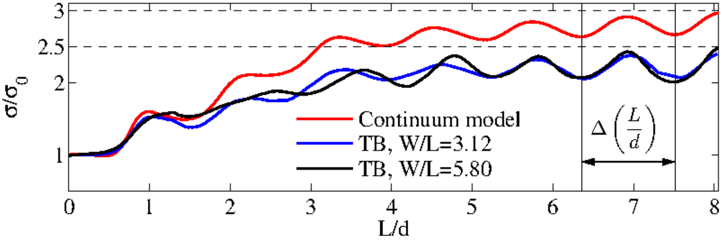
Armchair

Characteristic length scale for the RSO coupling: $d = \frac{\pi}{k_{SO}} \sim \frac{1}{\lambda^2}$ ← RSO coupling

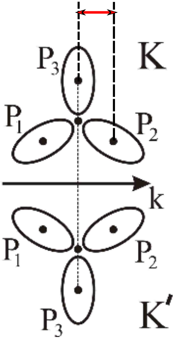
# Zig-zag and armchair edges
# Orientation of the pockets



# Oscillation in the conductivity for armchair edges




Armchair




accumlated phase of the electron bouncing between electrodes

$$\Delta\Phi = \pm m\sqrt{3}k_{SO}L = 2\pi \quad \text{where} \quad m = 0, 1, 2$$

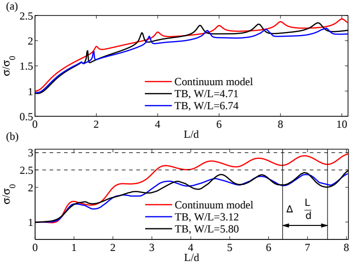
condition for constructive interference

$$\frac{L}{d} = \frac{2}{\sqrt{3}} \approx 1.15$$

where $d = \frac{\pi}{k_{SO}} \sim \frac{1}{\lambda^2}$

characteristic length scale for the RSO coupling

# Conductivity in TB model



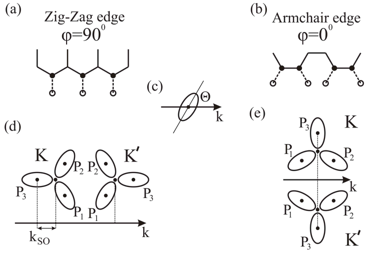

$\sigma = 7/3\sigma_0$ (TB & cont.)

Zig-zag

$\sigma = 3\sigma_0$ (cont.)

$\sigma = 5/2\sigma_0$ (TB)

Armchair




$$\sigma = n_C\sigma_C + \sum_i n_i\sigma(\Theta_i), \quad \text{where} \quad \sigma_C = \sigma_0/4$$

$$\sigma(\Theta_i) = \frac{v_a^2\cos^2(\Theta_i) + v_b^2\sin^2(\Theta_i)}{v_a v_b}\frac{\sigma_0}{4} \quad \text{and} \quad \sigma_0 = \frac{4}{\pi}\frac{e^2}{h}$$

J. Nilsson, A.H.Castro Neto, F. Guinea, N.M.R. Peres, Phys.Rev. B **78**, 045405 (2008).

Gy. Dávid,P . Rakyta, L. Oroszlány, J. Cserti, Phys.Rev.B **85**, 041402 (2012).

## Zig-zag

| pocket | C | $P_1$ | $P_3$ | $P_3$ |
|---|---|---|---|---|
| $\Theta_i$ (for $\mathbf{K}$) | - | $\frac{5\pi}{3}$ | $\frac{\pi}{3}$ | $\pi$ |
| $\Theta_i$ (for $\mathbf{K}'$) | - | $\frac{4\pi}{3}$ | $\frac{2\pi}{3}$ | 0 |
| $n_i$ | 0 | 2 | 2 | 0 |

$$\sigma = 7/3\sigma_0$$

## Armchair

| pocket | C | $P_1$ | $P_3$ | $P_3$ |
|---|---|---|---|---|
| $\Theta_i$ (for $\mathbf{K}$ and $\mathbf{K}'$) | - | $\frac{7\pi}{6}$ | $\frac{11\pi}{6}$ | $\frac{\pi}{2}$ |
| $n_i$ | 2 | 1 | 1 | 2 |

$$\sigma = 5/2\sigma_0$$

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András Pályi: Budapest University of Technology and Economics, Budapest, Hungary

- OTKA 108676

Publication:

Péter Rakyta, László Oroszlány, Andor Kormányos, József Cserti:
*Finite-size effects on the minimal conductivity in graphene with Rashba spin–orbit coupling*,
Physica E: Low-dimensional Systems and Nanostructures,  **75**, 1-6 (2016), January

a special volume on

“FRONTIERS IN QUANTUM ELECTRONIC TRANSPORT - IN MEMORY OF MARKUS BÜTTIKER”

to be published next year in Physica E.

We hope that our results are a tribute for the long lasting legacy of the simple yet powerful Landauer–Büttiker formalism and to the memory of Markus Büttiker.