

Landau levels and SdH oscillations in monolayer transition metal dichalcogenide semiconductors

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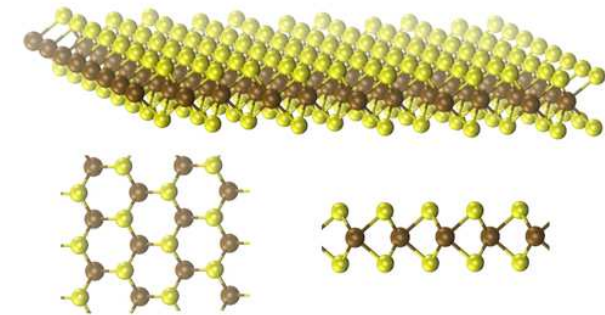
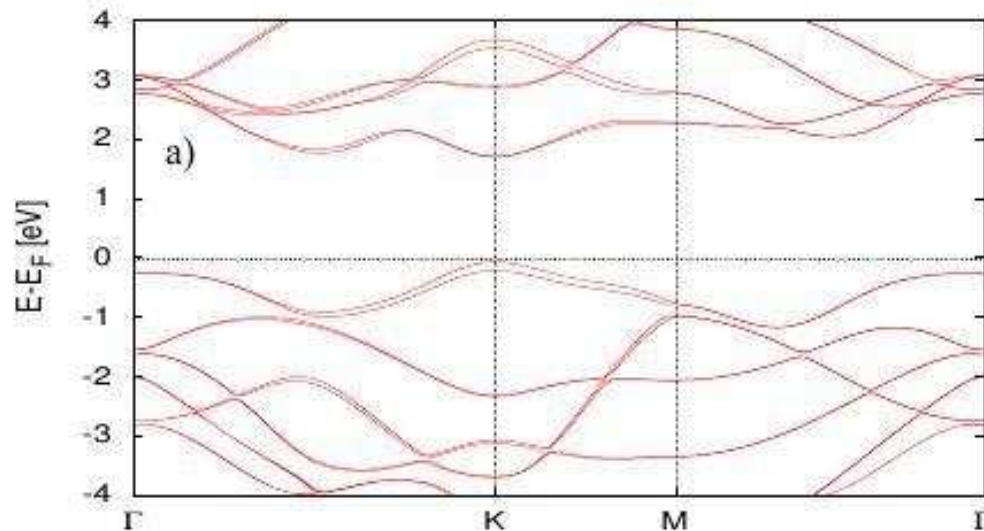
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Kormányos et. al., New Journal of Physics **17**, 103006 (2015).



Transition metal dichalcogenides semiconductors

- ▣▣▣▣ Top view: three-fold symmetry
- ▣▣▣▣ **No inversion symmetry.**
- ▣▣▣▣ Atomically thin material with direct band gap.
 $\sim 1 - 2$ eV. DFT, LSDA for MoS_2 :



(Nano-TCAD Group)

Kormányos et. al, Phys. Rev. B **88**, 045416 (2013).





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Two-band continuum model for valleys $\tau = \pm K$

seven-band model $\rightarrow \hbar \hat{q} = \mathbf{p} + \frac{e}{\hbar} \mathbf{A} \rightarrow$ two-band model by Löwdin partitioning

$$H_{\text{eff}}^{\tau} = H_0 + H_{\text{so}}^{\tau} + H_{\mathbf{k}\cdot\mathbf{p}}^{\tau}$$

$$H_0 = \frac{\hbar^2}{2m_e} \frac{\hat{q}_+ \hat{q}_- + \hat{q}_- \hat{q}_+}{2} + \frac{1}{2} g_e \mu_B B_z s_z \quad H_{\text{so}}^{\tau,s} = \begin{pmatrix} \tau \Delta_{\text{vb}} s_z & 0 \\ 0 & \tau \Delta_{\text{cb}} s_z \end{pmatrix}$$

$$H_{\mathbf{k}\cdot\mathbf{p}}^{\tau,s} = H_D^{\tau,s} + H_{\text{as}}^{\tau,s} + H_{3w}^{\tau,s} + H_{\text{cub}}^{\tau,s}$$

$$H_D^{\tau,s} = \begin{pmatrix} \varepsilon_{\text{vb}} & \tau \gamma_{\tau,s} \hat{q}_-^{\tau} \\ \tau \gamma_{\tau,s}^* \hat{q}_+^{\tau} & \varepsilon_{\text{cb}} \end{pmatrix}$$

$$H_{\text{as}}^{\tau,s} = \begin{pmatrix} \alpha_{\tau,s} \hat{q}_+^{\tau} \hat{q}_-^{\tau} & 0 \\ 0 & \beta_{\tau,s} \hat{q}_-^{\tau} \hat{q}_+^{\tau} \end{pmatrix}$$

$$H_{3w}^{\tau,s} = \begin{pmatrix} 0 & \kappa_{\tau,s} (\hat{q}_+^{\tau})^2 \\ \kappa_{\tau,s}^* (\hat{q}_-^{\tau})^2 & 0 \end{pmatrix},$$

$$H_{\text{cub},1}^{\tau,s} = \dots$$



Landau Levels (LLs)

- ▣ harmonic oscillator eigenfunctions as basis states: $\pi_- = \frac{2}{l_B}a$, $\pi_+ = \frac{2}{l_B}a^\dagger$, $l_B\sqrt{\hbar/eB}$.
- ▣ matrix representation of H_{eff}^τ on a finite subspace spanned by the basis functions.
- ▣ Landau Levels: eigenvalues of H_{eff}^τ .

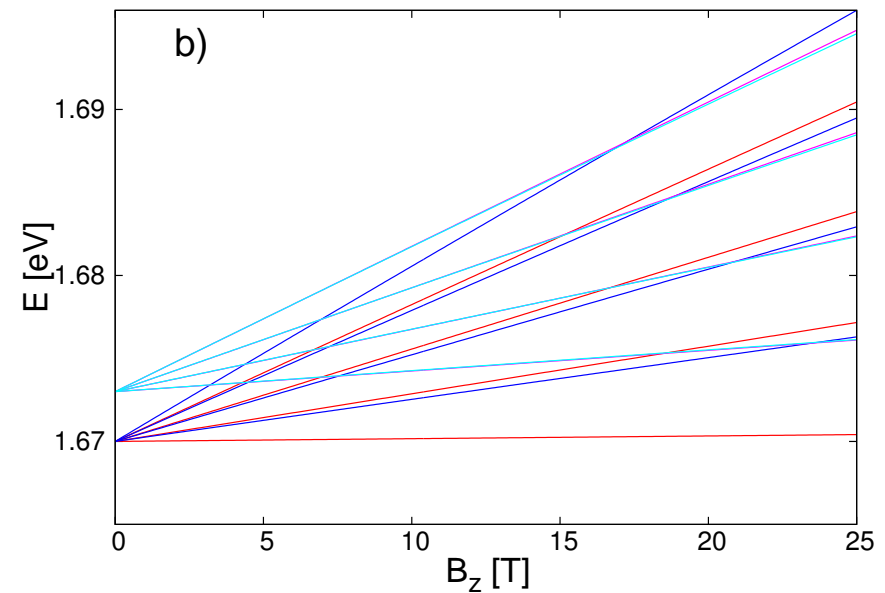
LLs in the conduction band:

spin \uparrow , K-valley

spin \uparrow , K'-valley

spin \downarrow , K-valley

spin \downarrow , K'-valley



Approximation of the LLs

- ▣▣▣▣ Neglecting the trigonal warping and cubic terms ($H_{3w}^{\tau,s}$, $H_{cub,1}^{\tau,s}$).
- ▣▣▣▣ Another Löwdin-partitioning to obtain effective **single-band** Hamiltonian separately for conduction and valence bands.
- ▣▣▣▣ Leading to a harmonic oscillator-like Hamiltonians:

$$E_{n,vb}^{\tau,s} = \varepsilon_{vb}^{\tau,s} + \hbar\omega_{vb}^{(\tau,s)} \left(n + \frac{1}{2} \right) + \frac{1}{2}g_e\mu_B B_z s + \frac{1}{2}g_{vl,vb}^{(s)}\mu_B B_z \tau,$$

$$E_{n,cb}^{\tau,s} = \varepsilon_{cb}^{\tau,s} + \hbar\omega_{cb}^{(\tau,s)} \left(n + \frac{1}{2} \right) + \frac{1}{2}g_e\mu_B B_z s + \frac{1}{2}g_{vl,cb}^{(s)}\mu_B B_z \tau.$$

- ▣▣▣▣ **valley splitting** linearly depends on B_z .



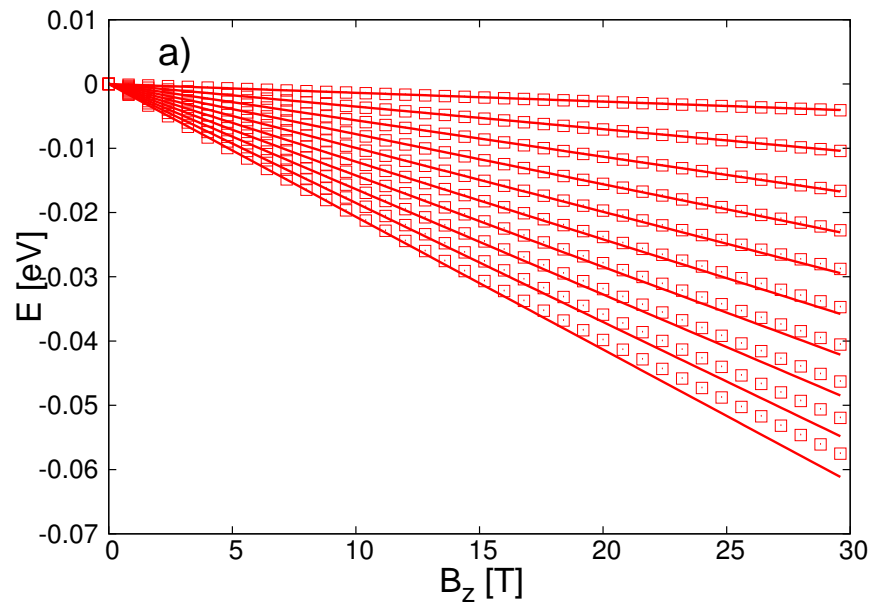


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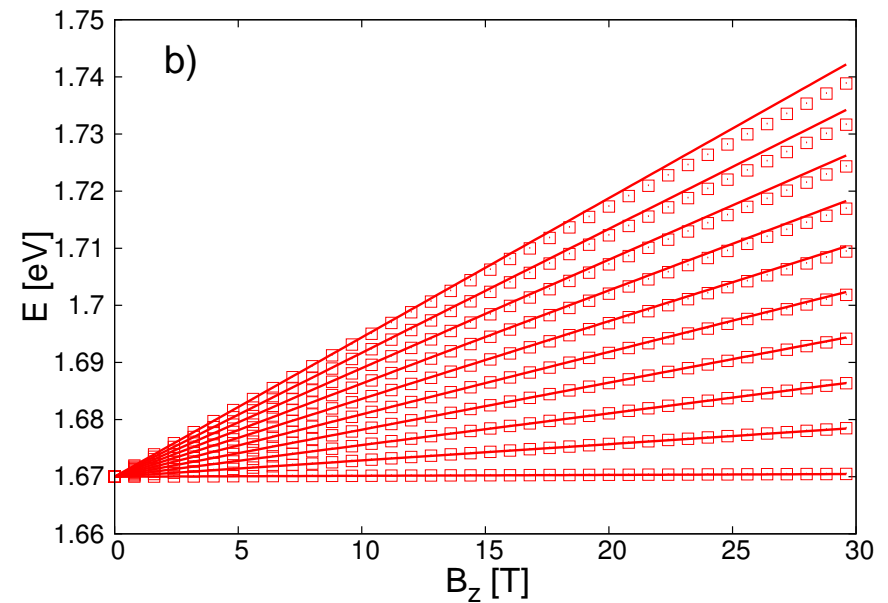
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Approximation vs full quantum
valley K, spin \downarrow

valence band



conduction band



➡ The trigonal warping is relevant for higher magnetic fields.

➡ At higher magnetic fields the valley splitting is not linear in B .





Calculation of the SdH oscillations

Calculation of the conductivity σ_{xx} :

- **neglecting the intra-valley scattering between the spin-split bands**: due to the specific form of the intrinsic SOC
- **neglecting the inter-valley scattering** in the absence of magnetic impurity: large momentum change, requires simultaneous spin-flip
- Considering the **intra-valley, intra-band** scatterings and short range scatterers.



Self-consistent Born approximation

random disorder potential $V(\mathbf{r})$ with short range correlations

$$\langle V(\mathbf{r})V(\mathbf{r}') \rangle = \lambda_{sc}\delta(\mathbf{r} - \mathbf{r}')$$

self-energy $\Sigma_R^{\tau,s} = \Sigma_r^{\tau,s} + i\Sigma_i^{\tau,s}$

$$\Sigma_r^{\tau,s} + i\Sigma_i^{\tau,s} = \frac{\lambda_{sc}}{2\pi l_B^2} \sum_{n=0}^{\infty} \frac{1}{E - E_n^{\tau,s} - (\Sigma_r^{\tau,s} + i\Sigma_i^{\tau,s})}$$

$E_n^{\tau,s}$ are the **approximated or exact** LL energies.

$\lambda_{sc} \sim$ scattering rate calculated by the Born-approximation in zero magnetic field.

Ando T, J. Phys. Soc. Jpn. **37**, 1233 (1974).



conductivity σ_{xx}

▣▣▣▣▣ Kubo-formalism:

$$\sigma_{xx}^{\tau,s} = \frac{e^2}{\pi^2 \hbar} \int dE \left(-\frac{\partial f(E)}{\partial E} \right) \sigma_{xx}^{\tau,s}(E)$$

$$\frac{\sigma_{xx}^{\tau,s}(E)}{(\hbar\omega_c^{(i)})^2} = \sum_{n=0}^{\infty} (n+1) \text{Re}[G_A^{\tau,s}(n, E)G_R^{\tau,s}(n+1, E) - G_A^{\tau,s}(n, E)G_A^{\tau,s}(n, E)]$$

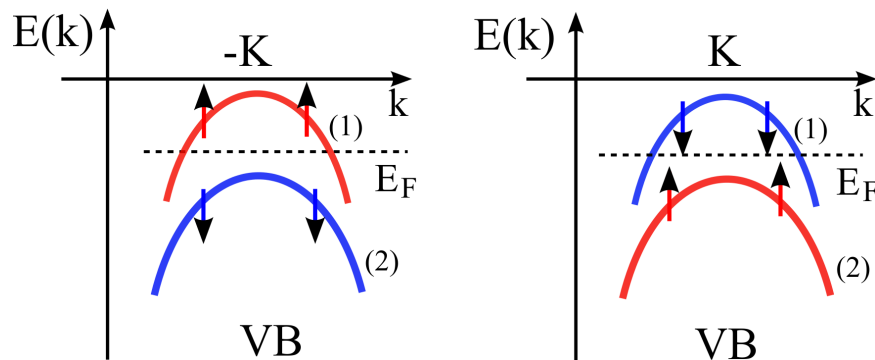
▣▣▣▣▣ $G_R^{\tau,s}(n, E)$ and $G_A^{\tau,s}(n, E)$ are the retarded and advanced Greens-functions:

$$G_{R,A}^{\tau,s}(n, E) = [E - E_n^{\tau,s} - \Sigma_{R,A}^{\tau,s}]^{-1}$$

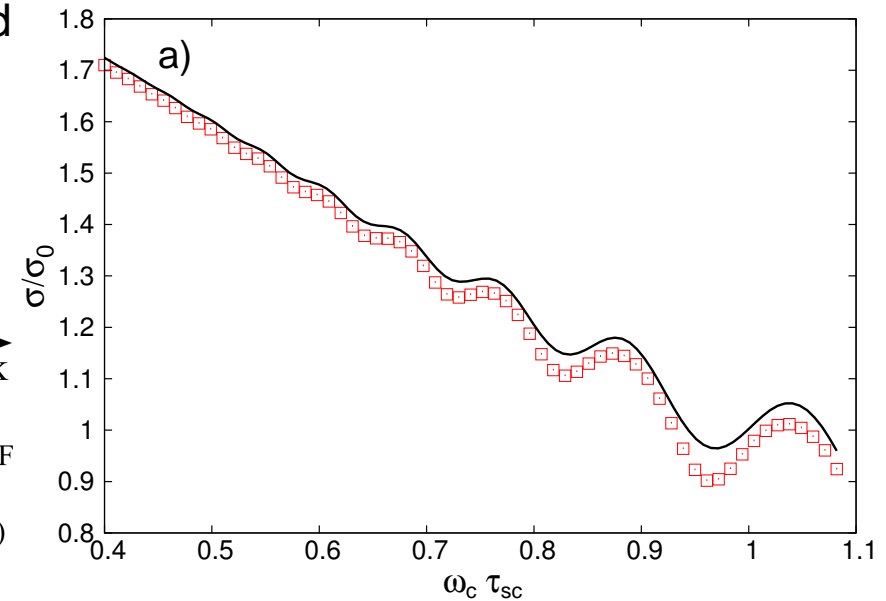


Calculated SdH oscillations for p-doped WSe₂

- ▮ The amplitudes of the oscillations are not captured well by the approximated LLs.
- ▮ Reason: few LLs under the Fermi level.



numerical SdH, and approximated SdH



Calculated SdH oscillations for n-doped MoS₂

- two bands contribute to

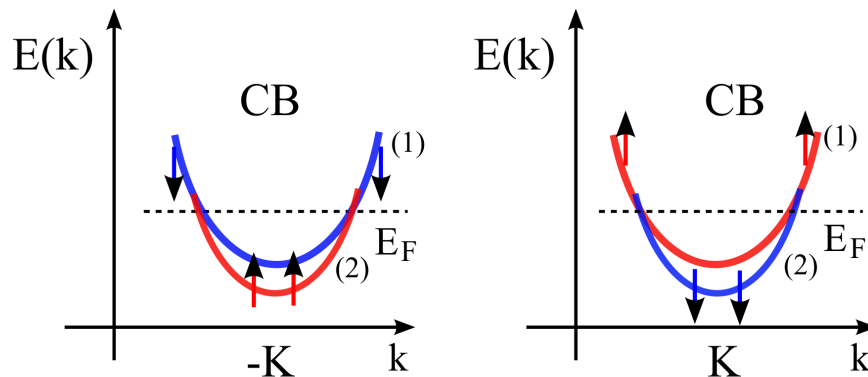
$$\sigma_{xx} = \sigma_{xx}^{(1)} + \sigma_{xx}^{(2)}$$

- parameters from GW calculations

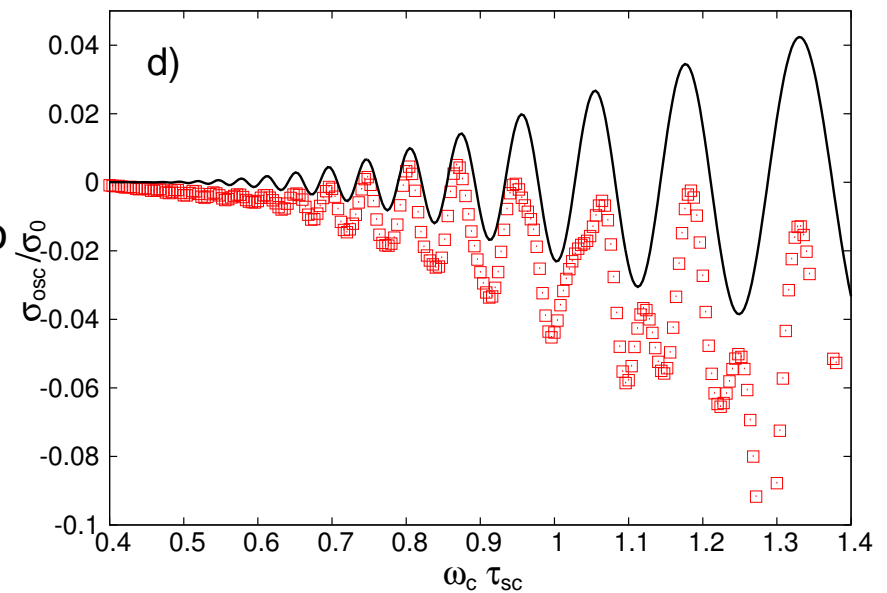
- complex oscillation pattern due to

valley splitting

(different oscillation periods in the two valleys)



numerical SdH, and approximated SdH



Comparing to experiments on MoS₂

- Fitting the **analytical expression** to the **experimental data** of X. Cui et. al., 2015 advance online publication in Nature Nanotechnology (arXiv:1412.5977)

experimental SdH, and approximated SdH

